EXERCISE – V

JEE PROBLEMS

1. (a) The curve described parametrically by, $x=t^2+t+1$, $y=t^2-t+1$ represents **[JEE 99, 2 + 2 + 2]**

(A) a parabola (B) an ellipse

(C) a hyperbola (D) a pair of straight lines

Sol.

(b) Let P (a sec θ, b tan θ) and Q (a sec ϕ, b tan ϕ), where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P & Q, then k is equal to

(A) $\frac{a^2+b^2}{a}$ (B) $-\left(\frac{a^2+b^2}{a}\right)$ (C) $\frac{a^2+b^2}{b}$ (D) $-\left(\frac{a^2+b^2}{b}\right)$

Sol.

(c) If x= 9 is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents, is

(A) $9x^2 - 8y^2 + 18x - 9 = 0$ (B) $9x^2 - 8y^2 - 18x + 9 = 0$

(C) $9x^2 - 8y^2 - 18x - 9 = 0$ (D) $9x^2 - 8y^2 + 18x + 9 = 0$

Sol.

2. The equation of the common tangent to the curve $y^2 = 8x$ and $xy = -1$ is **[JEE 2002 (Scr.)]**

(A) $3y = 9x + 2$ (B) $y = 2x + 1$

(C) $2y = x + 8$ (D) $y = x + 2$

Sol.

3. Given the family of hyperbolas $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ for $\alpha \in (0, \pi/2)$ which of the following does not change with varying α ? **[JEE 2003, (Scr.)]**

(A) abscissa of foci (B) eccentricity

(C) equations of directrices (D) abscissa of vertices

Sol.

4. The line $2x + \sqrt{6}y = 2$ is a tangent to the curve $x^2 - 2y^2 = 4$. The point of contact is **[JEE 2004 (Scr.)]**

(A) $\left(4, -\sqrt{6}\right)$ (B) $\left(7, -2\sqrt{6}\right)$ (C) (2, 3) (D) $\left(\sqrt{6}, 1\right)$

Sol.

5. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of midpoint of the chord of contact. **[JEE 2005 (Mains), 4]**

Sol.

6. If a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axis coincides with the major and minor axis of the ellipse, and product of their eccentricities is 1, then

(A) equation of hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(B) equation of hyperbola $\frac{x^2}{9} - \frac{x^2}{25} = 1$

(C) focus of hyperbola (5, 0)

(D) focus of hyperbola is $\left(5\sqrt{3}, 0\right)$ **[JEE 2006, 5]**

Sol.

Comprehension : (3 questions)

7. Let ABCD be a square of side length 2 units. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of the square ABCD. L is a line through A **[JEE 2006, 5 + 5 + 5]**

(a) If P is a point on C_1 and Q is another point on C_2, then $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to

(A) 0.75 (B) 1.25 (C) 1 (D) 0.5

Sol.

(b) A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is

(A) ellipse (B) hyperbola

(C) parabola (D) parts of straight line

Sol.

(c) A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts. M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is

(A) 1/2 sq. units (B) 2/3 sq. units

(C) 1 sq. unit (D) 2 sq. units

Sol.

8. (a) A hyperbola, having the transverse axis of length 2 sin θ, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is **[JEE 2007, 3 + 6]**

(A) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ (B) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

(C) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ (D) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Sol.

(b) Match the statements in **Column I** with the properties in **Column II**.

Column – I	**Column – II**
(A) Two intersecting circles	(P) have a common tangent
(B) Two mutually external circles	(Q) have a common normal
(C) Two circles, one strictly inside the other	(R) do not have a common tangent
(D) Two branches of a hyperbola	(S) do not have a common normal

Sol.

9. (a) Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents **[JEE 2008, 3 + 3]**

(A) four straight lines, when c = 0 and a, b are of the same sign.

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a.

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

Sol.

(b) Consider a branch of the hyperbola, $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1-\sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}}-1$ (C) $1+\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}}+1$

Sol.

10. Match the conics in column I with statements/ expressions in Column II. **[JEE 2009]**

Column–I	**Column–II**
(A) Circle	(P) The locus of the point (h, k) for which the line hx + ky= 1 touches the circle $x^2 + y^2 = 4$
(B) Parabola	(Q) Points z in the complex plane satisfying $\|z + 2\| - \|z - 2\| = \pm 3$
(C) Ellipse	(R) Points of the conic have parametric representation $x = \sqrt{3}(1-t^2/1+t^2)$, $y = 2t/1+t^2$
(D) Hyperbola	(S) The eccentricity of the conic lies in the interval $1 < x < \infty$
	(T) Points z in the complex plane satisfying $Re(z+1)^2 = \|z\|^2 + 1$

Sol.

11. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then **[JEE 2009]**

(A) equation of ellipse is $x^2 + 2y^2 = 2$

(B) the foci of ellipse are $(\pm 1, 0)$

(C) equation of ellipse is $x^2 + 2y^2 = 4$

(D) the foci of ellipse are $(\pm \sqrt{2}, 0)$

Sol.

Paragraph for Questions 12 to 13

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. **[JEE 2010]**

12. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}\, y - 20 = 0$ (B) $2x - \sqrt{5}\, y + 4 = 0$

(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Sol.

13. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$ (B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$ (D) $x^2 + y^2 - 24x - 12 = 0$

Sol.

14. The line 2x + y = 1 is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is **[JEE 2010]**

Sol.

15. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

Sol. **[JEE 2011]**

16. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$ **[JEE 2011]**

Sol.